Volume Integral Formulation Using Facet Elements For Electromagnetic Problem Considering Conductors And Dielectrics

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A new integral formulation is presented, enabling the computation of resistive, inductive and capacitive effects considering both conductors and dielectrics in the frequency-domain. The considered application here allows us to neglect any propagation effects and magnetic materials. In this paper, we will show how to improve the unstructured-PEEC (U-PEEC) approach to consider dielectric materials, keeping the same benefits. Results obtained with this formulation are compared to experimental data.

***Index Terms*—Volume Integral Equation, Dielectric materials, Unstructured-PEEC, Magneto-harmonic.**

I. Introduction

Electromagnetic problems coupled with external circuit can be efficiently solved using the partial element equivalent circuit (PEEC)[1]. It is an integral equation based method, transforming a meshed electromagnetic device into a circuit of lumped elements R-L-C and sources. The classical PEEC is limited, because only structured mesh are supported. This limitation has recently been overcome by using facets element[2] for general meshes. This major improvement enables the treatment of more complex geometries.

The PEEC is used for a wide range of frequency, from low-frequency application[3] to telecommunication devices as antennas[4]. Most of these applications only consider conductive materials, and deals with different type of geometrical regions (volume or surface).

The contribution of this paper is to extends the PEEC method taking into account capacitive effects in presence of dielectric materials. In this paper, the dielectric material is associated to a volume mesh. We also have developed a variant of the formulation where the dielectric mesh can be limited to its boundary. This alternative will be presented in the full paper.

II. Formulation

Let us define the following regions: Ω_J and Ω_D contains respectively the conductors and the dielectrics, and $\Omega = \Omega_J \cup \Omega_D$. Their borders are defined as $\Gamma_J = \partial\Omega_J$ and $\Gamma_D = \partial\Omega_D$.

Let us consider a problem with conductors, dielectrics and without any magnetic material. We have the following behavior laws

$$\begin{cases} \mathbf{J} = \sigma\mathbf{E} & \text{in } \Omega_J, \\ \mathbf{D} = \epsilon\mathbf{E} = \epsilon_0\mathbf{E} + \mathbf{P} & \text{in } \Omega_D, \\ \mathbf{B} = \mu_0\mathbf{H} \end{cases}$$

and according to Maxwell's equation we get

$$\mathrm{rot}\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} = -\mu_0\frac{\partial\mathbf{H}}{\partial t} \quad (1)$$

$$\mathrm{rot}\mathbf{H} = \mathbf{J} + \frac{\partial\mathbf{D}}{\partial t} = \left(\mathbf{J} + \frac{\partial\mathbf{P}}{\partial t}\right) + \epsilon_0\frac{\partial\mathbf{E}}{\partial t} \quad (2)$$

and thanks to the Lorentz gauge, we have

$$\mathbf{A} = \frac{\mu_0}{4\pi}\int_\Omega \frac{1}{r}\left(\mathbf{J} + \frac{\partial\mathbf{P}}{\partial t}\right)\mathrm{d}\Omega \quad (3)$$

$$\frac{\partial V}{\partial t} = \frac{1}{4\pi\epsilon_0}\int_\Omega \left(\mathbf{J} + \frac{\partial\mathbf{P}}{\partial t}\right)\boldsymbol{\nabla}\left(\frac{1}{r}\right)\mathrm{d}\Omega \quad (4)$$

where $\mathbf{A}$ and V are the magnetic vector potential and the scalar electric potential respectively.

The formulation limited to resistive and inductive conductors[2] uses $\mathbf{J}$ as unknown. Its solenoidality was imposed with a circuit solver. In this new formulation considering dielectrics, a new unknown $\mathbf{J_t}$ is choosen:

$$\mathbf{J_t} = \mathbf{J} + \frac{\partial\mathbf{D}}{\partial t} = (\sigma + j\omega\epsilon)\,\mathbf{E} \quad (5)$$

and thanks to (2), we have $\mathrm{div}\mathbf{J_t} = 0$ in the whole problem. Then, we are discretizing $\mathbf{J_t}$ using the same facet elements interpolation as in [2] to overcome the structured mesh restriction.

The integral equation consists in matching electrical behavior law everywhere in the materials. From (3) and (4), we can write

$$\mathbf{E}(\mathbf{P}) = -\frac{\partial\mathbf{A}}{\partial t}(\mathbf{P}) - \boldsymbol{\nabla}V(\mathbf{P}) \quad (6)$$

and using the behavior law defined in (5), we have in the frequency domain:

$$\frac{\mathbf{J_t}(\mathbf{P})}{\sigma + j\omega\epsilon} = -j\omega\mathbf{A}(\mathbf{P}) - \boldsymbol{\nabla}V(\mathbf{P}). \quad (7)$$

Thus, the use of a standard Galerkin projection in the frequency-domain leads to the following matrix system

$$([\mathbf{R_t}] + j\omega[\mathbf{L_t}])\,\{\mathbf{I}\} = \{\delta V\} \quad (8)$$

$$[\mathbf{R_t}]_{i,j} = \int_\Omega \frac{\mathbf{w}_i\mathbf{w}_j}{\sigma + j\omega\epsilon}\mathrm{d}\Omega \quad (9)$$

$$[\mathbf{L_t}]_{i,j} = j\omega\int_\Omega \mathbf{w}_i\int_\Omega \frac{\sigma + j\omega(\epsilon - \epsilon_0)}{\sigma + j\omega\epsilon}\frac{\mathbf{w}_j}{r}\mathrm{d}\Omega, \quad (10)$$

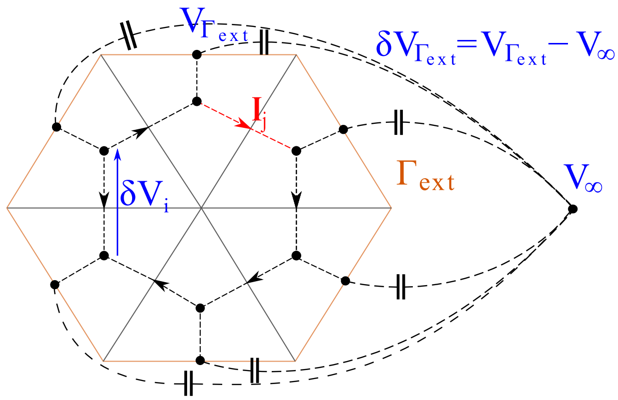

Fig. 1. Example of a primal and dual mesh with capacitive external branches.

with $[\mathbf{R_t}]$ and $[\mathbf{L_t}]$ are the resistive sparse-matrix and the inductive dense-matrix respectively. The right-hand-side vector is the voltage of the dual mesh branches (see Fig. 1), where the dual mesh represents the equivalent electrical circuit. We can notice that V_i is the averaged voltage on element i(or averaged voltage on border facets of Γ_{ext})[2]. $\mathbf{I}_j$ is the current flowing trough the facet j.

The equation (8) corresponds to incomplete circuit equations. Actually, the capacitive effects are yet not taken into account, so we have to add some capacitive branches linking the border facets of Γ_{ext} to a common node ∞, as shown on the Fig. 1. We suppose $V_\infty = 0$. So,

$$V_{\Gamma_{\text{ext}_i}} = \frac{1}{S_i} \int_{\Gamma_{\text{ext}_i}} V_i \mathrm{d}\Gamma$$

Using (4), we can express $V_{\Gamma_{\text{ext}}} = \frac{1}{4\pi\epsilon_0} \frac{1}{j\omega} [\mathbf{P_t}]\{\mathbf{I}\}$ with

$$[\mathbf{P_t}]_{i,j} = \int_{\Gamma_{\text{ext}_i}} \frac{1}{S_i} \int_\Omega \mathbf{w}_j \frac{\sigma + j\omega(\epsilon - \epsilon_0)}{\sigma + j\omega\epsilon_0} \boldsymbol{\nabla} \left(\frac{1}{r}\right) \mathrm{d}\Omega \mathrm{d}\Gamma \tag{11}$$

Finally, the equation to solve is

$$\left([\mathbf{R_t}] + j\omega[\mathbf{L_t}] + \frac{1}{j\omega}[\mathbf{P_t}]\right)\{\mathbf{I}\} = \{\delta V\}. \tag{12}$$

In practise, we use a circuit solver to ensure the current conservation. This solver does not need any geometrical information, so the conservation is ensured for any geometry and discretization.

For high frequencies, we used an equivalent thin shell model for the conductors[5] to facilitate the consideration of the skin effect in the thickness.

III. Numerical example

We have tested our formulation on a real device (see Fig. 2), made of two layers of copper $35\mu m$-thick separated by a layer of dielectric FR4 with a thickness of $1.47mm$ (Fig 2).

The device is meshed using $1,143$ hexahedrons which leads to $3,940$ degree of freedom. Here, the mesh is light enough to compute the solution using a direct solver. The Lower Upper (LU-factorization) solution takes about 15 seconds on a Dell Precision M4800 with an Intel(R) Core(TM) i7-4800MQ CPU @2.70GHz and 32Go RAM.

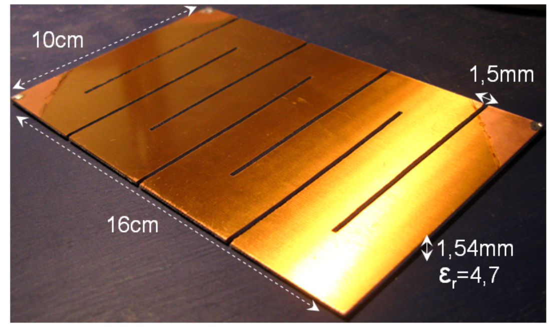

Fig. 2. Picture and dimensions of the studied device.

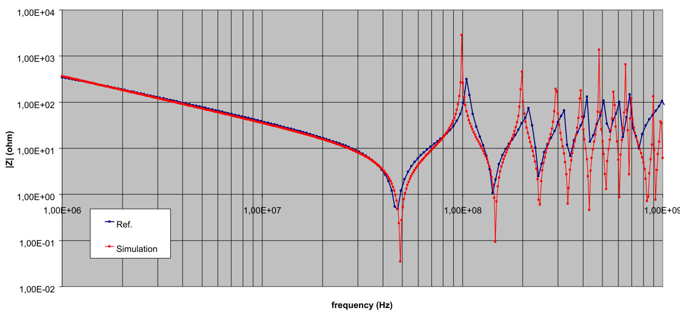

Fig. 3. Impedance modulus versus frequency. In blue: The reference (measurements), in red: the simulation results.

In addition, this formulation allows us to solve large scale problems using compression techniques, as the $\mathcal{H}$-matrices[6], on the dense integral matrices $[\mathbf{L_t}]$ and $[\mathbf{P_t}]$.

The impedance measurements without any connection between conductors (opened-circuit) and simulation are compared on the Fig. 3.

We can see that the frequency of almost all peaks match pretty well, until high frequency effects appear. On the other hand, the amplitude of the peaks does not match exactly.

IV. Conclusion

We have presented a new formulation to consider conductors and dielectrics with a very few restrictive hypothesis. Thanks to the circuit solver, it can be applied to any geometry and external electrical circuit can easily be added.

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